A rebuttal to Ajay Sharma's paper on $E=mc^2$

In a seminal paper published in 1905[1] Einstein produced a very original derivation of the variation of mass depending of the variation of a body's energy. The conclusion of Einstein's paper[1] is that "If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2". Einstein derives this very interesting relationship by applying the results of his other 1905 seminal paper[2] to a body that emits[2] "two plane waves of light" of equal energy L, arranged to be at 180^0 with respect to each other such that there is no recoil for the emitting body after the radiation was emitted. In the following note we will refute the errors in the Ajay Sharma[3] paper that claims that the result of electromagnetic radiation can be "mass increase" under certain circumstances (see equation (13) starting in section 1.3.1 page 201). In the following we will be using Einstein notation employed in [1].

The Mass Variation and the Recoil Energy

As in the original Einstein paper[1], we will consider a body at rest in an inertial frame S that emits only one light wave of energy L in a direction making an angle ϕ with the positive x axis. The results will be analyzed from the inertial frame S' in relative motion with speed v with respect to S.
According to [2], in S':

$$L' = L\gamma(1 - \beta\cos\phi) \quad (1.1)$$

hence:

$$H_0 = H_1 + L' = H_1 + L\gamma(1 - \beta\cos\phi)\ , \quad (1.2)$$

H_0 and H_1 are the energy of the body in S', before and after the emission, v is the relative speed between S and S' and

$$\beta = \frac{v}{c} \quad \text{and} \quad \gamma = 1/\sqrt{1-\beta^2}$$

The conservation of energy in S' gives:

$$H_0 - H_1 = L\gamma(1 - \beta\cos\phi) \quad (1.3)$$

The conservation of energy in S gives:

$$E_0 - E_1 = L \quad (1.4)$$

where E_0 and E_1 are the energy of the body in S', before and after the emission .Therefore the kinetic energy variation is:

K_0-K_1=(H_0-E_0)-(H_1-E_1)=(H_0-H_1)-(E_0-E_1)=Lγ(1 − β cosϕ)-L=

$$=L(\gamma-1)-L\gamma\beta\cos\phi \approx \frac{L/c^2 * v^2}{2} - L\gamma\beta\cos\phi \qquad (1.5)$$

or:

$$K_1 - K_0 \approx -\frac{L/c^2 * v^2}{2} + L\gamma\beta\cos\phi \qquad (1.6)$$

The above shows that a body emitting the radiation energy L experiences a mass decrease by $\frac{L}{c^2}$ and a kinetic energy increase due to recoil Lγβ cosϕ.

By contrast, Ajay Sharma obtains the following nonsensical set of equations , starting at page 201:

$$H_0 = H_1 + \beta L - \beta L\frac{v}{c}\cos\theta \qquad \text{(Ajay\_1)}$$

above β is actually γ , making the situation a little confusing. Ajay Sharma also confuses the total energy H with the kinetic energy K. From (Ajay_1), Sharma gets the trivial expression:

$$H_1 - H_0 = -\beta L + \beta L\frac{v}{c}\cos\theta \qquad \text{(Ajay\_2)}$$

From (Ajay_2) Sharma concludes that:

$$\Delta m = -\frac{L}{cv}\cos\theta + \frac{L}{c^2} \qquad \text{(eq 13 , top of page 201)}$$

Sharma's eq 13 contains several gross errors because:

1. Instead of doing the correct thing in calculating the variation of kinetic energy $K_1 - K_0 \approx -\frac{L/c^2 * v^2}{2} + L\gamma\beta\cos\phi$, Ajay Sharma inadvertently calculates the variation $H_1 - H_0 = -\beta L + \beta L\frac{v}{c}\cos\theta$ which is NOT the variation of kinetic energy.

2. As we have seen in (1.6) only the term $-\frac{L}{c^2}$ is attributable to mass variation, the other term is the recoil kinetic energy, Sharma incorrectly rolls both terms into Δm

3. The next worst error is that the incorrect Sharma derivation comes up with an inverted sign for the mass variation term concluding that there is "mass increase" as a resultant of electromagnetic radiation (see the "$+\frac{L}{c^2}$" term instead of the correct term $-\frac{L}{c^2}$) when in reality, the mass clearly decreases as a result of radiating light

Conclusion

Einstein's derivation stands.
Ajay Sharma's derivation is refuted.

References

1) A. Einstein "Does the Inertia of a Body Depend upon its Energy-Content", Annalen der Physik **18**, 1905

2) A. Einstein "On the Electrodynamics of Moving Bodies", Annalen der Physik **17**, 1905

3) A. Sharma,"The Origin of Generalized Mass-Energy Equation its applications in General physics and Cosmology", Physics Essays, 17 (2004) 195-222.